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Effect of orthogonal planes on inelastic torsional response

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ABSTRACT

The torsional stiffness has important influence on the seismic response of an asymmetric structure, both in the elastic and inelastic range. For elastic structures, it is immaterial whether the stiffness is provided solely by planes parallel to the direction of earthquake or by both parallel and orthogonal planes. The issue of how the relative contribution of orthogonal planes affects the torsional response of inelastic structures is examined here, through analytical studies on the response of asymmetric single storey building models. It is shown that the torsional response of inelastic structures is affected primarily by the total torsional stiffness, and not so much by whether such stiffness is contributed solely by parallel planes or by both parallel and orthogonal planes.

INTRODUCTION

An asymmetric building structure is defined as one in which the centres of resistance do not coincide with the centres of mass. When subjected to earthquake ground motion, such a structure undergoes both translational and rotational motion even when the earthquake excitation is purely translational. This coupling between translation and torsion may significantly magnify the displacements and forces induced in certain elements.

A number of parameters govern the response of asymmetric buildings, but one that has the most significant effect is the torsional stiffness. All resisting planes, both parallel and perpendicular to the earthquake motion, contribute to the torsional stiffness. It is therefore apparent that the planes that are perpendicular to the earthquake motion, referred to herein as orthogonal planes, must be included in the analytical model used to study the earthquake response of asymmetric buildings.

A number of researchers have studied the elastic and inelastic torsional response of single-storey and multi-storey building models. However, certain questions, particularly those related to the inelastic torsional behavior, have not been adequately addressed. One such issue is the effect of orthogonal elements on the inelastic torsional behavior. Recent studies on torsional response and the effect of orthogonal planes include those by Correnza et al (1994), De La Llera and Chopra (1994) Paulay (1996), and Humar and Kumar (1999a, 1999b). Many of the conclusions arrived at in these studies are at variance with each other, adding to the confusions surrounding the issue.

On the basis of analytical studies of elastic and inelastic response Humar and Kumar (199a, 1999b) concluded that the single most important parameter governing the torsional response was the ratio of the uncoupled torsional frequency to the uncoupled translational frequency, or equivalently, the ratio of torsional to translational stiffness. For elastic structures it was immaterial whether the torsional stiffness was provided only by the planes parallel to the direction of the earthquake, or by both the parallel and the orthogonal planes. There was evidence to show that this was true also for structures strained into the inelastic range. The present study explores this further by addressing the following issues: (1) effect of the variation in torsional stiffness of orthogonal planes while the overall torsional stiffness of the system is held constant, (2) effect of yielding in the orthogonal planes on the torsional behavior of the system, and (3) the effect of the uncoupled translational period of the building model studied.

A majority of the results presented here are obtained from the linear and nonlinear dynamic analysis of a single storey, mono-symmetric building model for its response to a set of 12 earthquake motions.

BUILDING MODELS STUDIED

The mono-symmetric, single storey building model with rigid diaphragm used in this study is shown in Fig. 1. The model, which is rectangular in plan, has three resisting planes parallel to the y-axis and two planes parallel to

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the x-axis. The distribution of stiffness among the y-direction planes is such that the centre of stiffness is eccentric with respect to the geometric centre and lies at a distance e from the latter. The planes parallel to the x-axis are identical and are symmetrically placed. The dimension of the building along the x-axis is b and along the y-axis is a. The plan aspect ratio is defined as $\alpha = a/b$. The floor mass is assumed to be uniformly distributed, and the mass radius of gyration about the geometric centroid is denoted by r. The uncoupled translational frequency ω_y and the uncoupled torsional frequency ω_θ are defined as

$$\omega_y = \sqrt{\frac{K_y}{m}} \tag{1a}$$

$$\omega_\theta = \sqrt{\frac{K_{\theta R}}{mr^2}} \tag{1b}$$

where K_y is the total stiffness of planes in the y-direction and $K_{\theta R}$ is the torsional stiffness about the centre of stiffness, which is also the centre of resistance (CR). The uncoupled frequency ratio is defined as

$$\Omega_R = \omega_\theta/\omega_y = \sqrt{\frac{K_{\theta R}}{r^2 K_y}} \tag{2}$$

For the purpose of obtaining numerical results the following data is used for all the models studied: b = 36 m, m = 400 t, mr^2 = 5400 t.m^2. A wide range of building models with different combination of e/b and Ω_R values is selected for study.

To study the effect of orthogonal planes a parameter γ is introduced. It is defined as the ratio of the torsional stiffness (about CR) of planes parallel to y-axis to the overall torsional stiffness of the system, $K_{\theta R}$. A lower value of γ indicates a higher contribution from orthogonal elements towards the total torsional stiffness of the model.

EARTHQUAKE MOTIONS

A set of 12 earthquake records, each having ground acceleration data in two orthogonal horizontal directions, is selected. For the selected records the average value of the ratio of peak ground acceleration in the minor direction component of an earthquake to that in the major direction component is found to be 0.88. In the dynamic time history analysis carried out in this study, the major direction component of each earthquake is scaled to a peak ground acceleration of $0.3g$, while the minor direction component is scaled to a peak ground acceleration of $0.88 * 0.3g = 0.264g$.

To obtain the design elastic spectrum, each of the 12 major component records is scaled so that its peak acceleration is $0.3g$. The average of the elastic spectra for the scaled records and for 5% damping is taken as the design response spectrum. The same design spectrum is used for both x and y directions as is the standard practice in seismic design.

DETAILS OF ANALYSIS

In the torsionally unbalanced (TUB) model of the building, the mass centre (CM) coincides with the geometric centre of the building. The stiffnesses of the individual planes in the model are determined once the values of the eccentricity e/b, the frequency ratio Ω_R and the period of the building have been selected. For the presentation of analytical results, a reference torsionally balanced (TB) model is defined as the one having the same T_y, Ω_R and the location of CR as the TUB model, but with the CM shifted to coincide with CR.

The force-displacement relationship for each resisting plane in both the TB and TUB models is assumed to be bi-linear, with a post-yield stiffness equal to 5% of the initial elastic stiffness. In the TB model, the elastic design strength in the y-direction, V_{ey}, is obtained from the design response spectrum for the translational period of vibration T_y. The total yield strength of the TB model in the y-direction is now taken as $V_{y0} = V_{ey}/R$, where the force modification factor R is equal to 4. This value of R is based on a ductility capacity $\mu = 4$ and the assumption that the maximum total displacements imposed by the design earthquake in both the elastic and the inelastic systems is the same. The yield strengths f_i of individual planes in the TB model are proportional to their stiffness k_i.

An equivalent static method based on the concept of design eccentricity is used to obtain the yield strengths of planes in the y-direction of TUB model. The following design eccentricities suggested recently by Humar and

Kumar (199a, 1999b) are used

$$e_{d1} = e + 0.1b \tag{3}$$

$$e_{d2} = e - 0.1b \qquad \Omega_R \geq 1.0 \tag{4a}$$

$$e_{d2} = -0.1b \qquad \Omega_R < 1.0 \tag{4b}$$

The properties of the x-direction planes in the TB and TUB models are identical. The total stiffness of these planes is determined from the selected x translation period. This stiffness is equally divided among the two planes. The total elastic strength of elements in the x-direction, V_{ex}, is obtained from the design response spectrum corresponding to the x-direction translation period. The total yield strength in the x-direction is taken as $V_{ex}/4$ and each plane is assigned half of this strength.

Dynamic time history analyses are carried out on both the TB and TUB models for the earthquake records scaled as described earlier. It should be noted that the design eccentricities given by Eqs. 3 and 4 include a provision for possible accidental torsion. In order to verify whether these provisions lead to adequate design, the dynamic analyses carried out on the building models must also include the effect of accidental eccentricity. Recently De La Llera and Chopra (1994) have suggested that the effect of accidental eccentricity can reasonably be taken into account by shifting the mass centres by $\pm 0.05b$ in the models being analysed. The TUB models are therefore modified by moving the centre of mass $\pm 0.05b$ along the x axis, and the higher of the two responses obtained from corresponding modified models is considered for each element.

In the analysis of each pair of TUB models, it is assumed that earthquake excitation is applied simultaneously along both x and y-directions. To demonstrate the effect of yielding in orthogonal elements on torsional response of the system, the above mentioned set of analyses is repeated with the difference that the strength of orthogonal planes is taken to be very high. This ensures that the orthogonal planes remain elastic, providing a basis for comparison between the response of a model with orthogonal planes that remain elastic and a model with orthogonal planes that may yield.

The maximum ductility demand in a resisting plane in any torsionally unbalanced model subjected to a given earthquake is denoted by μ_u while the maximum ductility demand for the associated torsionally balanced model is denoted by μ_b. The ratio of ductilities $r_\mu = \mu_u/\mu_b$ provides a measure of the effect of torsional motion. A mean value of the ductility ratios, obtained for the set of 12 earthquakes, is denoted by $\bar{r}_\mu$

RESULTS OF RESPONSE ANALYSES

Results are presented here for building models with the uncoupled period of translation in y-direction equal to 1.0 s., and that in the x-direction equal to 0.5 s.

The mean ductility ratio for the flexible edge element $\bar{r}_{\mu f}$ is obtained for a range of values of e/b, Ω_R and γ. For brevity, only some of the results are presented in Fig. 2. It should be noted that in Fig. 2, $\gamma = 1$ represents a building model without orthogonal elements. Also plotted in Fig. 2 are the responses of building models for which orthogonal planes are designed to remain elastic. The value of $\bar{r}_{\mu f}$ is less than 1 in all cases implying that the ductility demand on the flexible edge plane in a torsionally unbalanced building, designed according to the procedure suggested here, is substantially less than that in the associated balanced building.

The mean value of ductility ratio for the stiff edge element $\bar{r}_{\mu s}$ is plotted against e/b in Fig. 3 for selected values of γ and Ω_R. Ductility ratio $\bar{r}_{\mu s}$ is also found to be less than 1 for all the models studied, except when $\Omega_R = 1.0$. Even for $\Omega_R = 1$, the ductility ratio is no more than 10% higher than 1.

Effect of orthogonal planes

Results of analysis including those shown in Fig. 2 indicate that the presence of orthogonal planes usually reduces the ductility demand at the flexible edge of the building. In general, for a smaller value of γ, i.e. for a higher contribution of orthogonal planes, the reduction is higher. However, the reduction in ductility demands on account of the presence of orthogonal planes is quite modest (< 10%) and may be considered insignificant for practical purposes. This observation holds for all values of γ.

Results of analysis including those shown in Fig. 3 indicate that the presence of orthogonal planes that may yield increases the ductility demand at the stiff edge of the building for $\Omega_R = 1$ but reduces this ductility demand for other values of Ω_R. For a lower value of γ, this effect is more pronounced. Here again, the difference in ductility demands for models with different γ values is quite small ($< 10\%$) and may be considered insignificant.

The results obtained in this study tend to indicate that for a single storey building system in which orthogonal planes as well as parallel planes yield during an earthquake, the ductility demand of an edge plane depends more or less on the total torsional stiffness of the building and not on what part of it is contributed by orthogonal planes. The effect of γ on the inelastic torsional response of single storey building models is thus quite small.

Effect of yielding in orthogonal planes

It will be noted that the decrease in the ductility demand on the flexible edge planes due to the presence of orthogonal becomes more significant when such planes remain elastic. On the other hand, elastic orthogonal planes generally cause an increase in the ductility demand on stiff edge planes. This can be explained as follows. The total response of a lateral load-resisting plane in a single storey building model is a combination of rotational and lateral responses. The rotational motion adds to the lateral displacement at the flexible edge but compensates the lateral displacement at the stiff edge. Except for systems that are torsionally very flexible ($\Omega_R < 0.75$), the total displacement at the stiff edge reduces as a result of torsion. If the orthogonal planes remain elastic, they provide a relatively higher torsional resistance in comparison to the case when these orthogonal planes are yielding. As a consequence, the presence of elastic orthogonal planes reduces the torsional response of the system. The result is a reduction in the total response at the flexible edge and an increase in the total response at the stiff edge.

Analysis similar to those described in the previous paragraphs are repeated for models with different y-direction periods. The results, not presented here, indicate a pattern very similar to the one seen in Figs. 2 and 3. All the conclusions drawn from the results of previous analyses hold true for irrespective of the period of the building model analysed.

CONCLUSIONS

The torsional behavior of asymmetric buildings subjected to earthquake motion is strongly influenced by the torsional stiffness as measured by the ratio of uncoupled rotational frequency to the uncoupled translational frequency. The torsional stiffness may arise from the planes parallel to the direction of earthquake, or as is most often the case, is a sum of contributions from planes both parallel and perpendicular to the direction of earthquake. When the torsional stiffness is contributed partly by the orthogonal planes, the ductility demand in the flexible planes is reduced, though not by a large amount. On the other hand, the ductility demand in the stiff edge plane may be reduced or increased depending upon the value of the frequency ratio. In all cases, the reduction or increase is fairly moderate.

The trends noted above are accentuated when the orthogonal planes stay elastic during the earthquake motion. The flexible edge ductility demands decrease further, while the stiff edge ductility demands increase.

The influence of orthogonal planes on ductility demands, as noted in the previous paragraphs is consistent for all periods.

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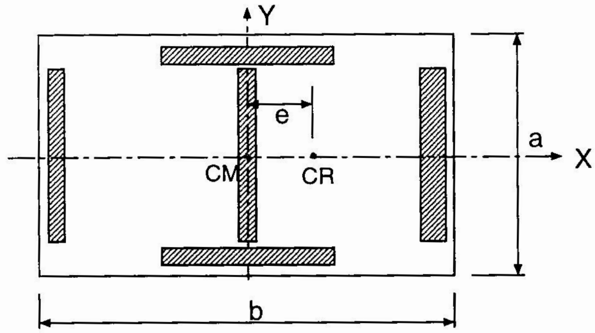

Fig. 1: Plan view of a 5-plane single storey monosymmetric building model

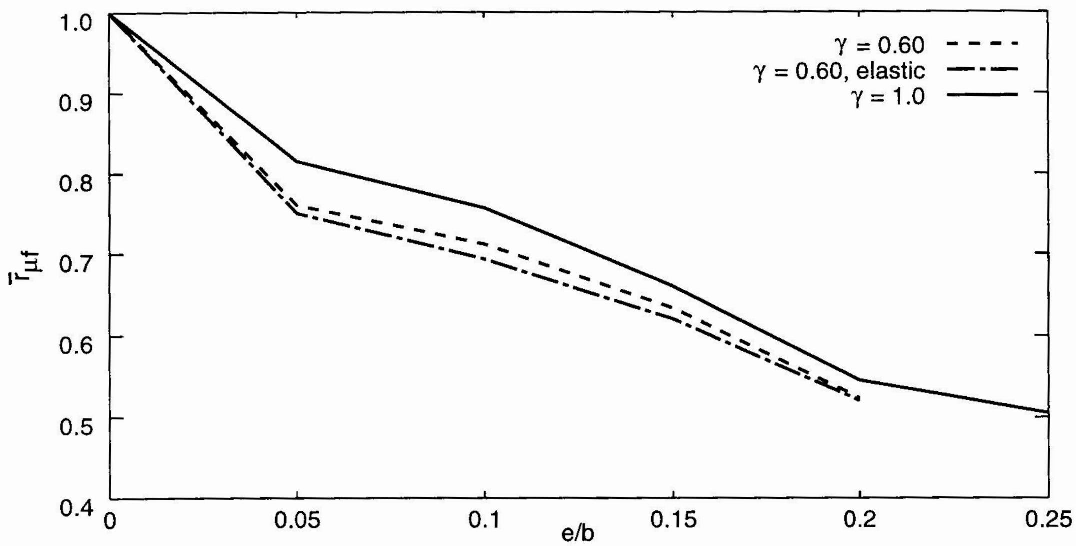

Fig. 2a: Ratio of flexible edge ductility demands, $\Omega_R = 1.0$

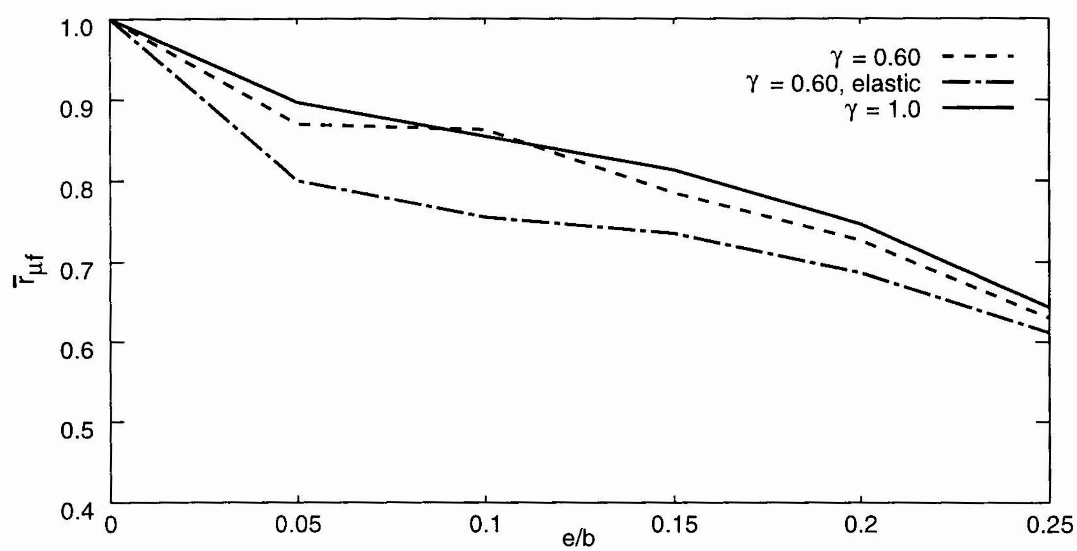

Fig. 2b: Ratio of flexible edge ductility demands, $\Omega_R = 1.25$

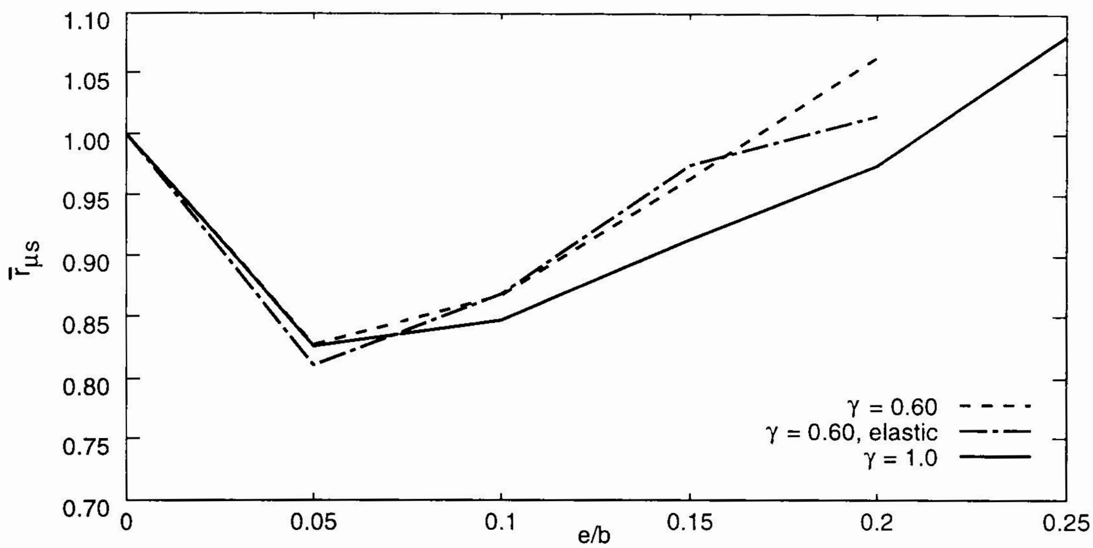

Fig. 3a: Ratio of stiff edge ductility demands, $\Omega_R = 1.0$

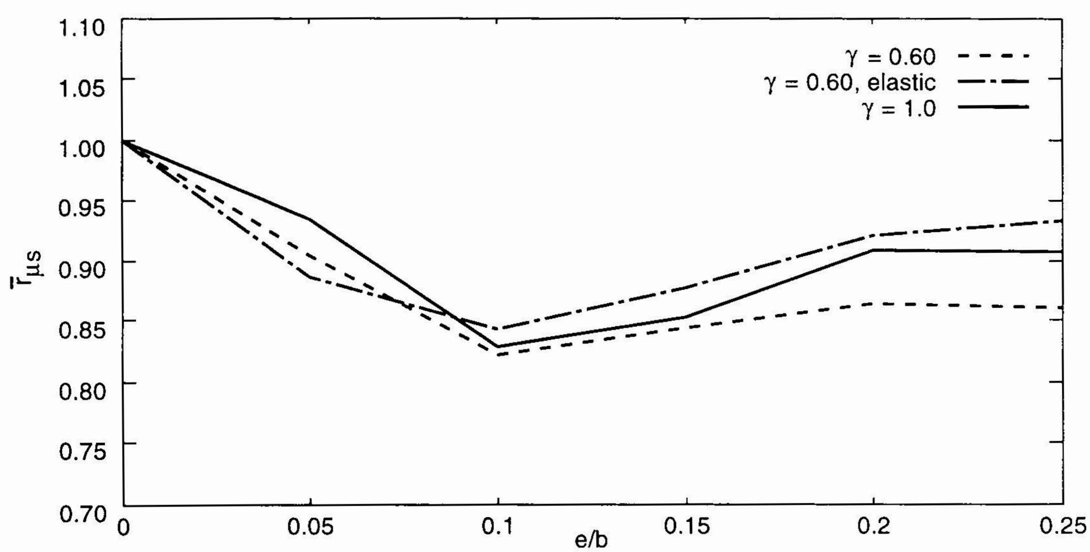

Fig. 3b: Ratio of stiff edge ductility demands, $\Omega_R = 1.25$